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OPTIMIZED RECOVERY OF DAMAGED ELECTRICAL POWER GRIDS

by

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March 2006

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This thesis formulates and solves a mixed-integer program to plan the recovery of an electrical power transmission grid that has been damaged by a natural disaster or terrorist attack. The damage can be extensive and recovery can take weeks or months. An efficient recovery plan that maximizes the utilization of repair resources can help ensure swift restoration of services.

The network recovery-planning model is implemented in GAMS (General Algebraic Modeling System) and uses CPLEX as the solver. An electrical grid based on IEEE's 300-bus transmission network is used for testing. To simulate varying degrees of damage to the network, we choose up to 20% of the grid's lines, buses and transformers to be placed out of service. Based on the availability of repair resources, the repair-time horizon and penalties for unserved demand, the model produces a repair schedule that minimizes the cost of power shed.

We demonstrate that for a network with up to 8% of its components damaged, the model can produce an optimal recovery plan within 20 minutes on a 2 GHz personal computer. For our largest test-case with 20% of network components damaged, the recovery plan is within 7% of optimal after 1 hour of solver time.

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OPTIMIZED RECOVERY OF DAMAGED ELECTRICAL POWER GRIDS

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EXECUTIVE SUMMARY

This thesis formulates and solves a mixed-integer program (MIP) to plan the recovery of an electrical power transmission grid that has been damaged.

Electrical power grids span large geographical areas and, consequently, are hard to protect against malicious activities; they are also vulnerable to natural disasters. The formulation of a recovery plan after catastrophic and widespread damage to the power grid can be a complicated process. The complexity can be exacerbated by pressure on planning staff to produce a plan quickly. Decisions on which parts of the grid to repair in what order must be made while considering the availability of repair resources and spares, and the relative importance of customers affected by the service disruption.

We develop a MIP called RECovery OPtimization (RECOP) to aid in the recovery-planning process. RECOP accounts for the availability of repair resources and spares for damaged grid components, and schedules their allocation in order to obtain a recovery schedule that minimizes the economic cost of unmet demand for energy. If necessary, precedence relationships enforcing the relative orders in which components need to be repaired can be defined as part of the inputs to RECOP. These relationships can be used to reflect a requirement to restore certain sectors of the network first.

We implement RECOP in the General Algebraic Modeling System (GAMS) and use CPLEX as the solver. An electrical grid based on IEEE's 300-bus transmission network is used for testing. To simulate varying degrees of damage to the network, we choose up to 20% of the grid's buses, lines and transformers to be placed out of service. We demonstrate that for a network with up to 8% of its components damaged, the model can produce an optimal recovery plan within 20 minutes on a 2 GHz personal computer. For our largest test-case with 20% of network components damaged, the recovery plan is within 7% of optimal after 1 hour of solver time.

We also explore the use of Benders decomposition to solve RECOP, and the use of a heuristic to solve RECOP, at least approximately. In all our test cases, Benders decomposition takes at least as much time as the MIP requires to produce a solution of

the same or inferior quality. For the terrorist-attack scenarios, where relatively few components are damaged, the heuristic solutions are up to 40% worse in terms of energy shed compared to the MIP solutions. The difference in performance between the two solution methods narrows in hurricane-damage scenarios, however, where more components are damaged. In these cases, each heuristic solution obtained in ten seconds is within 10% of the solution obtained by solving the MIP directly for an hour. This shows the potential of the heuristic solution method for larger-sized problems to obtain "reasonable" solutions within a short period of time.

We demonstrate that, although the number of components of an electrical grid that is damaged during a coordinated terrorist attack may be relatively small compared to the number damaged during a hurricane, the impact in terms of energy shed and economic costs could be larger. This is because terrorists can selectively target critical components that cannot be replaced immediately due to delivery and manufacturing lead-times for replacement parts. This highlights the potential impact of spares availability on grid recovery and underscores the need to keep a strategic inventory of critical components to facilitate quick restoration of the power grid. To this end, RECOP can be used as a tool to help evaluate the effectiveness of various mixes of inventories.

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I. INTRODUCTION

A. BACKGROUND

This thesis formulates and solves a mixed-integer program (MIP) to plan the recovery of an electrical power transmission grid that has been damaged by a natural disaster or terrorist attack. The recovery plan takes into account the scarcity of repair resources and spares, and schedules their allocation in order to minimize the economic cost due to unmet energy demand. Alternative solution methods using heuristics and decomposition are also presented.

Infrastructure networks, e.g., road, electricity and telecommunications networks, inherently span large geographical areas. Consequently, they are hard to protect and can be vulnerable to natural disasters, e.g., hurricanes and earthquakes, and to man-made attacks, e.g., terrorist attacks and war. One way to reduce a network's vulnerability is to harden critical components. Salmeron et al. [2004] develop bilevel mathematical models that can be used to identify critical sets of a power grid's components, such as generators, transmission lines and transformers; these components become prime candidates for hardening.

However, if a power grid has already been damaged, this question arises: How should repair be carried out to minimize disruption of services to customers? The benefits of an efficient recovery process for the electric power grid cannot be over-stated due to the dependency of other industries on electrical power. The then-chairman of the Federal Reserve Board, Alan Greenspan, commented in November 2005 about the effects of Hurricane Katrina: "The combination of flooding, wind damage, and a lack of electric power also forced many crude oil refineries and natural gas processing plants to shut down" [Federal Reserve Board 2005]. A quick recovery from a blackout can minimize the impact on the public and the economy.

Power-grid recovery planning is a process that involves power-system modeling and "strategy planning." Power-system modeling ensures that the proposed recovery process does not inadvertently throw the system back into collapse due to unusual system

conditions and phenomena. This thesis focuses on strategy-planning, which guides the formulation of a recovery process in order to meet a given objective. Examples of such objectives include minimizing the total amount of energy shed, minimizing the duration of the restoration process or "priority restoration" of critical customers such as those belonging to the health- and public-services sectors.

The formulation of a recovery plan after catastrophic and widespread damage to an electrical power grid is a complicated process. The complexity can be exacerbated by pressure on planning staff to produce a plan quickly. Decisions on the order in which repairs must be made have to take into account the repair-equipment and manpower resources available and the objective of the restoration. This is a dynamic problem as repair crews can move in from unaffected areas to help in the recovery work, or move out in response to greater needs elsewhere.

An optimization-based decision-support system can help in scheduling the recovery plan. It can schedule repairs in order to minimize power-shedding over time, thereby limiting the impact of damage to the transmission grid on the grid's customers. It can also be used in a dynamic environment to track repairs and resources, and to reschedule them as new information becomes available.

Non-trivial, real-world scenarios exist where such a tool may prove useful. For example, Hurricane Charley caused the Florida Power and Light Company to mobilize 11,200 workers and replace over 5,100 low-voltage transformers and 900 miles of wire [Florida Power and Light Company 2004.] More recently, Hurricane Rita knocked out 82% and 38% of transmission lines in southeastern Texas and southwestern Louisiana, respectively [Entergy Corporation 2005.] Admittedly, these real-world problems are large in comparison with the 300-bus transmission network [IEEE 1993] used for testing our model. However, we show that our methods do solve problems defined on this 300-bus network in a reasonable amount of time: Thus, they provide a starting point for research into solving larger problems.

In contrast to storm damage, a coordinated terrorist attack on an electrical grid may see more limited damage that may, nonetheless, be highly disruptive if the terrorists

can identify and destroy critical components. Such components may include high-voltage transformers for which utilities usually keep few or no spares, because of their high cost and nominally high reliability. Long delivery or manufacturing lead times for these components, which may stretch over weeks or months, exacerbates the potential for disruption. Even if a national strategic inventory of critical, components having long replacement lead times is established [Electric Power Research Institute 2003], a shortage or delay in the delivery of replacement components cannot be ruled out. Under this scenario, our recovery optimization model can help to determine which facilities to repair first while awaiting delivery of additional spares necessary for full network recovery.

The model is intended as a high-level, low-resolution recovery-planning tool. It is not designed to produce detailed, hourly schedules for each individual resource or piece of equipment involved in the recovery exercise. Rather, the model prioritizes the repair of damaged components to ensure quick recovery while ensuring that resources are available to carry out the repair.

Existing literature on infrastructure network recovery planning is limited. Feng and Wang [2003] describe an integer-programming model for scheduling the emergency repair of highways damaged by an earthquake given that repairs are constrained by time and resources. The model focuses on "reachability" for rescue within the first three days after an earthquake occurs. It also considers the minimization of the risk from after-shocks on work-crews by the appropriate choice of a minimum-risk path. No network flow-balance or road-capacity constraints are considered. Barbarosoglu and Arda [2004] formulate a multi-commodity, multi-modal time-space network model to optimize the routing and scheduling of emergency-response vehicles in disaster-affected areas. While this model considers the traffic-handling capacity of an area's roads, it does not deal with the scheduling of the repair of these roads. Brown and Vassiliou [1993] introduce a decision-support system which uses optimization methods, simulation, and the decision-maker's judgment to assign tasks and resources for the repair of public works damaged by an earthquake. Only a simple, pure assignment model is used by that system.

B. THESIS OUTLINE

Subsequent chapters in this thesis are organized as follows: Chapter II describes the mathematical formulation of the network recovery planning model. Chapter III reports results of the model applied to the IEEE 300-bus transmission network. Attempts to speed up solution times using heuristics and Benders decomposition are also described. Section IV provides conclusions and points out areas for future research. Appendix A describes the changes made to the IEEE transmission-network's generation capacity to adapt it for our application. Appendices B and C present the data used in our hurricane-damage and terrorist-attack scenarios, respectively. Finally, Appendix D presents the algorithm used to implement the Benders decomposition of RECOP.

II. A MODEL FOR THE OPTIMIZED RECOVERY OF DAMAGED ELECTRICAL GRIDS

A. OVERVIEW

This chapter introduces the RECOP model (RECovery OPtimization) developed to optimize recovery of a damaged electrical power grid. The model starts with the premise that some of the power grid's components, e.g., buses, transmission lines, transformers, are damaged, resulting in power-shedding at various parts of the network. The objective of the model is to formulate a recovery plan that minimizes the monetary opportunity cost arising from this unmet demand over the repair-time horizon.

The rate of network recovery depends on the number of grid components that are damaged and the size of the available pool of various types of repair resources and replacement components. Each type of repair resource may be viewed as a type of work team, defined by characteristics such as the number and capability of the team's workers, and the tools and machinery available to them. Each unit of a repair resource is therefore a re-useable asset that can be deployed repeatedly to repair damaged components. Replacement components are spare parts which, by their nature, can be used only once. (These are typically critical components such as high-voltage transformers, which may have long replacement lead-times.)

The recovery process is very dynamic; the "health" of the grid progressively improves as more components are repaired, and the availability of repair resources and replacement parts can change over time. As such, the model's repair-time horizon is sub-divided into a series of discrete time periods. Repairs start at the beginning of a time period with a corresponding draw-down of applicable repair resources and replacement components. Each repair is assumed to take an integral number of time periods after which the repair resources are released for further repairs.

The number of time periods and their duration are scenario-dependent. For example, a model dealing with the scheduling of work-crew for the repair of downed transmission lines may use 12-hour time-periods with a total of 28 periods to reflect a planning horizon of two weeks.

If necessary, precedence relationships enforcing the relative orders in which components need to be repaired can be defined as part of the inputs to the model. Such relationships can be used to reflect a requirement to restore certain sectors of the network first. This is the case, for example, when a sector supports other critical infrastructure such as public-health and sanitation services, or when damage to road infrastructure forces work-crews to concentrate their initial repair efforts on accessible parts of the network.

The model output consists of a repair schedule by component and time period, resource utilization (parts and manpower), and the power flow and power shed in the grid at each stage of the recovery process.

B. RECOVERY OPTIMIZATION MODEL

RECOP is a linear MIP. Its core consists of a power-flow model that approximates the non-linear behavior of active power flows in a power grid. This approximation is adequate for a high-level, low-resolution model such as ours [Wood and Wollenberg 1996, p. 419.] For simplicity, we suppose that all loads are held constant over time: The model could be extended to incorporate load-duration curves through the introduction of partitioned time periods with distinct loads, but this is not explored here. Other parts of the RECOP model implement features described in Section A above. Additional modeling assumptions are discussed in Section C. The RECOP model follows:

Indices and Index Sets:

C	set of customer classes, $c \in C$
T	set of time periods, $t \in T = \{1, 2, ... \| T \|\}$
N	set of buses $i, j \in N$
$N' \subset N$	subset of buses that are damaged
$N'' \subset N'$	subset of buses that are damaged and need spares for repair
$N^d \subset N$	subset of buses that have a local load

$N^s \subset N$ subset of buses that have a local generation capability

$A \subset N \times N$ set of lines (this set includes transformers, which are modeled as lines in RECOP.)

$A' \subset A$ subset of lines that are damaged

$A'' \subset A'$ subset of lines that are damaged and need spares for repair

R set of repair resources, $r \in R$

R' set of spares, $r' \in R'$

$R'_i \subset R'$ set of spares needed to repair bus $i \in N''$

$R'_{i,j} \subset R'$ set of spares needed to repair line $i, j \in A''$

$P^{AA} \subset A' \times A'$ pairs of damaged lines $[(i, j), (i', j')]$ where line (i, j) must be repaired before starting repair on line (i', j')

$P^{AN} \subset A' \times N'$ pairs of damaged lines and buses $[(i, j), i']$ where line (i, j) must be repaired before starting repair on bus i'

$P^{NA} \subset N' \times A'$ pairs of damaged buses and lines $[i, (i', j')]$ where bus i must be repaired before starting repair on line (i', j')

$P^{NN} \subset N' \times N'$ pairs of damaged buses $[i, i']$ where bus i must be repaired before starting repair on bus i'

Parameters and "(units)", if applicable:

h duration (hours) of each time period t

ρ weight ($) for penalizing the objective function value if damaged buses or lines are not repaired

$\Delta r_{r,t}$ change in the quantity of type r repair resource available at the beginning of time t

$\Delta r_{r',t}$	change in the quantity of type r' replacement component available at the beginning of time t
$d_{i,c}$	load at bus $i \in N^d$ from customer of class $c \in C$ (MW)
I_i^d	$\begin{cases} 1 \text{ if } i \in N^d, \\ 0 \text{ otherwise} \end{cases}$
I_i^s	$\begin{cases} 1 \text{ if } i \in N^s, \\ 0 \text{ otherwise} \end{cases}$
$I_{i,j}^i$	$\begin{cases} 1 \text{ if } i \in N' \text{ and } (i,j) \in A, \\ 0 \text{ otherwise} \end{cases}$
$I_{i,j}^j$	$\begin{cases} 1 \text{ if } j \in N' \text{ and } (i,j) \in A, \\ 0 \text{ otherwise} \end{cases}$
$I_{i,j}$	$\begin{cases} 1 \text{ if } (i,j) \in A', \\ 0 \text{ otherwise} \end{cases}$
$u_{i,j}$	transmission capacity for line $(i,j) \in A$ (MW)
$v_{i,j,t',t,r}^n$	for bus $i \in N'$ where $(i,j) \in A$, $v_{i,j,t',t,r}^n$ equals $u_{i,j}$ if repair that started on the bus during time period t' has been completed by period t (MW), $t' \le t$. $v_{i,j,t',t,r}^n$ equals zero if bus $i \in N'$ is awaiting or under repair. Similarly for bus $j \in N'$, where $(i,j) \in A$.
$v_{i,j,t',t,r}$	for line $(i,j) \in A'$, $v_{i,j,t',t,r}$ equals $u_{i,j}$ during time period t if repair that started on the line during period t' has been completed by time t (MW), $t' \le t$.
$f_{i,t',t,r}^n$	for bus $i \in N'$, $f_{i,t',t,r}^n$ equals the number of units of type r resource used to repair the bus during time period t if the repair effort started during period t', $t' \le t$.

$f_{i,j,t',t,r}$ for line $(i,j) \in A'$, $f_{i,j,t',t,r}$ equals the number of units of type r resource used to repair the line during time period t if the repair effort started during period t', $t' \le t$.

$\tau^n_{i,r}$ for bus $i \in N'$, $\tau^n_{i,r}$ equals the number of time periods required to repair the bus using repair resource $r \in R$

$\tau_{i,j,r}$ for line $(i,j) \in A'$, $\tau_{i,j,r}$ equals the number of time periods required to repair the line using repair resource $r \in R$

$\lambda^n_{i,r'}$ for bus $i \in N''$, $\lambda^n_{i,r'}$ equals the number of units of type r' spare needed to repair the bus, $r' \in R'_i$

$\lambda_{i,j,r'}$ for line $(i,j) \in A''$, $\lambda_{i,j,r'}$ equals the number of units of type r' spare needed to repair the line, $r' \in R'_{i,j}$

t^n_i earliest time period to start repair on damaged bus $i \in N'$

$t_{i,j}$ earliest time period to start repair on damaged line $(i,j) \in A'$

$p^-_{i,c,t}$ unit cost of energy shed at node $i \in N^s$ incurred by customer of class $c \in C$ during time period t (\$/MWh)

$B_{i,j}$ susceptance of line $(i,j) \in A$ (per unit) derived from $\dfrac{\sigma_{i,j}}{\varphi^2_{i,j} + \sigma^2_{i,j}}$ where $\sigma_{i,j}$ and $\varphi_{i,j}$ are the per unit reactance and resistance of line $(i,j) \in A$, respectively

u^g_i generation capacity at bus $i \in N^s$ (MW)

M defining $\overline{P}$ and $\overline{B}$ as the maximum line-capacity and line-susceptance, respectively, and $\pi =$ 3.142 radians, then $M = \overline{P} + \overline{B}\pi$

Decision variables and "(units)", where applicable:

$y_{i,j,t}$ power flow from bus i to bus j during time period t (MW)

$x^n_{i,t,r}$ $\begin{cases} 1 \text{ if repair on bus } i \in N' \text{ starts during time period } t \text{ using resouce } r \in R, \\ 0 \text{ otherwise} \end{cases}$

$x_{i,j,t,r}$ $\begin{cases} 1 \text{ if repair on line } (i,j) \in A' \text{ starts during time period } t \text{ using resource} \\ \quad r \in R, \\ 0 \text{ otherwise} \end{cases}$

$q^n_{i,t}$ $\begin{cases} 1 \text{ if repair on bus } i \in N' \text{ has been completed during or before} \\ \quad \text{time period } t, \\ 0 \text{ otherwise} \end{cases}$

$q_{i,j,t}$ $\begin{cases} 1 \text{ if repair on line } (i,j) \in A' \text{ has been completed during or before} \\ \quad \text{time period } t, \\ 0 \text{ otherwise} \end{cases}$

$s^-_{i,c,t}$ load shed at bus $i \in N^d$ by customer of class $c \in C$ during time period t (MW)

$g_{i,t}$ power generation at bus $i \in N^s$ during time period t (MW)

$\theta_{i,t}$ phase angle at bus $i \in N$ during time period t (radians)

Formulation: (Remark: All units above are converted into per-unit values for a base load of 100 MW.)

$$\text{RECOP:} \min_{\mathbf{x,q,y,s,\theta,g}} \sum_{i \in N^d} \sum_{c \in C} \sum_{t \in T} hp^-_{i,c,t} s^-_{i,c,t} - \frac{\rho}{|T|} \sum_{i \in N'} \sum_{t \in T} q^n_{i,t} - \frac{\rho}{|T|} \sum_{(i,j) \in A'} \sum_{t \in T} q_{i,j,t} \quad (1)$$

s.t.

Power-flow balance:

$$\sum_{(j,i) \in A} y_{j,i,t} - \sum_{(i,j) \in A} y_{i,j,t} + \sum_{c \in C} I^d_i s^-_{i,c,t} + I^g_i g_{i,t} = \sum_{c \in C} d_{i,c} \quad \forall i \in N,\ t \in T \quad (2)$$

Phase-angle constraints:

$$y_{i,j,t} - B_{i,j}(\theta_{i,t} - \theta_{j,t}) \le M\left\{ I^i_{i,j}(1-q^n_{i,t}) + I^j_{i,j}(1-q^n_{j,t}) + I_{i,j}(1-q_{i,j,t}) \right\}$$
$$\forall (i,j) \in A,\ t \in T \qquad (3)$$

$$y_{i,j,t} - B_{i,j}(\theta_{i,t} - \theta_{j,t}) \ge -M\left\{ I^i_{i,j}(1-q^n_{i,t}) + I^j_{i,j}(1-q^n_{j,t}) + I_{i,j}(1-q_{i,j,t}) \right\}$$
$$\forall (i,j) \in A,\ t \in T \qquad (4)$$

Capacity of damaged arcs:

$$y_{i,j,t} \le \sum_{t' \in T | t' \le t} \sum_{r \in R} v_{i,j,t',t,r} x_{i,j,t',r} \quad \forall (i,j) \in A',\ t \in T \qquad (5)$$

$$y_{i,j,t} \ge -\sum_{t' \in T | t' \le t} \sum_{r \in R} v_{i,j,t',t,r} x_{i,j,t',r} \quad \forall (i,j) \in A',\ t \in T \qquad (6)$$

Capacity of arcs whose origin nodes *i* have been damaged:

$$y_{i,j,t} \le \sum_{t' \in T | t' \le t} \sum_{r \in R} v^n_{i,j,t',t,r} x^n_{i,t',r} \quad \forall (i,j) \in A,\ i \in N',\ t \in T \qquad (7)$$

$$y_{i,j,t} \ge -\sum_{t' \in T | t' \le t} \sum_{r \in R} v^n_{i,j,t',t,r} x^n_{i,t',r} \quad \forall (i,j) \in A,\ i \in N',\ t \in T \qquad (8)$$

Capacity of arcs whose destination nodes *j* have been damaged:

$$y_{i,j,t} \le \sum_{t' \in T | t' \le t} \sum_{r \in R} v^n_{i,j,t',t,r} x^n_{j,t',r} \quad \forall (i,j) \in A,\ j \in N',\ t \in T \qquad (9)$$

$$y_{i,j,t} \ge -\sum_{t' \in T | t' \le t} \sum_{r \in R} v^n_{i,j,t',t,r} x^n_{j,t',r} \quad \forall (i,j) \in A,\ j \in N',\ t \in T \qquad (10)$$

Capacity of undamaged arcs:

$$-u_{i,j} \le y_{i,j,t} \le u_{i,j} \quad \forall (i,j) \in A \setminus A',\ t \in T \qquad (11)$$

Maximum generating-unit output if connecting buses are undamaged:

$$0 \le g_{i,t} \le u^g_i \quad \forall i \in (N^s \cap N \setminus N'),\ t \in T \qquad (12)$$

Maximum generating-unit output if connecting buses are damaged:

$$0 \le g_{i,t} \le u^g_i q^n_{i,t} \quad \forall i \in (N^s \cap N'),\ t \in T \qquad (13)$$

Force binary variables $q^n_{i,t}$ and $q_{i,j,t}$ to be zero if damaged buses and lines, respectively, have yet to be repaired at time *t*:

$$q^n_{i,t} = \sum_{t' \in T | t' \le t - \tau^n_{i,r}} \sum_{r \in R} x^n_{i,t',r} \ \forall i \in N',\ t \in T \tag{14}$$

$$q_{i,j,t} = \sum_{t' \in T | t' \le t - \tau_{i,j,r}} \sum_{r \in R} x_{i,j,t',r} \quad \forall (i,j) \in A',\ t \in T \tag{15}$$

Repair-resource constraints:

$$\sum_{(i,j) \in A'} \sum_{t' \in T | t' \le t} f_{i,j,t',t,r} x_{i,j,t',r} + \sum_{i \in N'} \sum_{t' \in T | t' \le t} f^n_{i,t',t,r} x^n_{i,t',r} \le \sum_{t' \in T | t' \le t} \Delta r_{r,t'} \quad \forall r \in R,\ t \in T \tag{16}$$

Spare constraints:

$$\sum_{(i,j) \in A'' | r' \in R'_{i,j}} \lambda_{i,j,r'} \sum_{t' \in T | t' \le t} \sum_{r \in R} x_{i,j,t',r} + \sum_{i \in N'' | r' \in R'_i} \lambda^n_{i,r'} \sum_{t' \in T | t' \le t} \sum_{r \in R} x^n_{i,t',r} \le \sum_{t' \in T | t' \le t} \Delta r_{r',t'} \ \ \forall r' \in R',\ t \in T \tag{17}$$

Repair line (i,j) before line (i',j'):

$$\sum_{t' \in T | t' \le t} \sum_{r \in R} x_{i',j',t',r} \le \sum_{r \in R} \sum_{t' \in T | t' \le (t - \tau_{i,j,r})} x_{i,j,t',r} \ \ \forall [(i,j),(i',j')] \in P^{AA},\ t \in T \tag{18}$$

Repair line (i,j) before bus i':

$$\sum_{t' \in T | t' \le t} \sum_{r \in R} x^n_{i',t',r} \le \sum_{r \in R} \sum_{t' \in T | t' \le (t - \tau_{i,j,r})} x_{i,j,t',r} \ \ \forall [(i,j),i'] \in P^{AN},\ t \in T \tag{19}$$

Repair bus *i* before line (i',j'):

$$\sum_{t' \in T | t' \le t} \sum_{r \in R} x_{i',j',t',r} \le \sum_{r \in R} \sum_{t' \in T | t' \le (t - \tau^n_{i,r})} x^n_{i,t',r} \ \ \forall [i,(i',j')] \in P^{NA},\ t \in T \tag{20}$$

Repair bus *i* before bus i':

$$\sum_{t' \in T | t' \le t} \sum_{r \in R} x^n_{i,t',r} \le \sum_{r \in R} \sum_{t' \in T | t' \le (t - \tau^n_{i,r})} x^n_{i,t',r} \ \ \forall [i,i'] \in P^{NN},\ t \in T \tag{21}$$

Repair each damaged bus and line at most once:

$$\sum_{t\in T}\sum_{r\in R} x_{i,t,r}^{n} \le 1 \quad \forall i \in N' \tag{22}$$

$$\sum_{t\in T}\sum_{r\in R} x_{i,j,t,r} \le 1 \quad \forall (i,j) \in A' \tag{23}$$

Earliest time period to start repair of damaged buses and lines respectively:

$$x_{i,t,r}^{n} = 0 \quad \forall i \in N', r \in R, t \in T \mid t < t_i^n \tag{24}$$

$$x_{i,j,t,r} = 0 \quad \forall (i,j) \in A', r \in R, t \in T \mid t < t_{i,j} \tag{25}$$

Power shed must not exceed demand:

$$0 \le s_{i,c,t}^{-} \le d_{i,c} \quad \forall i \in N^d,\ c \in C,\ t \in T \tag{26}$$

Binary variables:

$$x_{i,t,r}^{n} \in \{0,1\} \quad \forall i \in N', r \in R, t \in T \tag{27}$$

$$x_{i,j,t,r} \in \{0,1\} \quad \forall (i,j) \in A', r \in R, t \in T \tag{28}$$

$$q_{i,t}^{n} \in \{0,1\} \quad \forall i \in N',\ t \in T \tag{29}$$

$$q_{i,j,t} \in \{0,1\} \quad \forall (i,j) \in A',\ t \in T \tag{30}$$

C. MODELING ASSUMPTIONS AND COMMENTS

1. Optimal Dispatch of Power

The RECOP power-flow model approximates an optimal dispatch of power through the electrical grid during each time period, i.e., the dispatch that a system operator would make given complete data on the state of repair in the network.

2. Objective Function

We suppose that power-shedding costs are much higher than generation costs and have therefore ignored generation costs in the objective function.

Most power grids have some level of redundancy meaning that during the normal operation of the grid, a few downed components may not lead to power-shedding. To ensure that all damaged buses and lines are repaired even if their restoration to service has no impact on the amount of power shed, we include the following penalty terms in the objective function:

$$-\frac{\rho}{|T|}\sum_{i\in N'}\sum_{t\in T} q_{i,t}^{n} - \frac{\rho}{|T|}\sum_{(i,j)\in A'}\sum_{t\in T} q_{i,j,t}$$

3. Admittance Constraints

Admittance (phase-angle) constraints should only be enforced for closed lines (i.e., lines connected to the system), which mandates that the lines and their associated end-buses are in working condition. These requirements are implemented by constraints (3) and (4). Lines that are working have the right-hand-side of constraints (3) and (4) set to zero. If a line or either of its two end-buses are damaged and have yet to be repaired, the upper and lower bound limits on (3) and (4) effectively drop the phase-angle constraints.

4. Power-Shedding

The power-flow model assumes that power-shedding ($s_{i,c,t}^{-}$) can range from 0 MW to the nominal demand for power, i.e., there is no restriction on the minimum viable supply to a load. We assume that this could be achieved by implementing selective or rolling-blackouts to the individual classes of customers that comprise the load, so that power-flow balance is always maintained in the grid.

5. Repair-Resource and Spares

Travel times for repair-resources (e.g., work crew) between repair locations are ignored, i.e., when repairs on a grid component are completed at the end of a time period, the associated repair resources are immediately available at the beginning of the next time period for the repair of any component. Similarly, transportation times for spares to reach their intended destinations are assumed to be zero. However, since transportation

of some components (such as large-sized high-voltage transformers) can take a long time, we assume this effect is accounted for in the delivery lead-time of these components.

6. Bus, Line and Transformer Capacity during Repair

Buses and lines that are damaged or under repair have zero capacity. They revert to their nominal capacities at the beginning of the time period immediately following the time period during which repairs are completed.

All lines that are connected to a damaged bus have zero power flow through them until the bus has been repaired.

D. IEEE 300-BUS TEST NETWORK DATA

This section describes the assumptions and modifications necessary to adapt the IEEE 300-bus network data for testing RECOP.

1. Network Generation Capacity

The total load (which is assumed to be constant over time) in the original IEEE test network exceeds generation capacity by 46 MW. To present a more realistic test case for RECOP, additional generation capacities are added to the system so that total generation capacity exceeds total load by 15%; the changes to the generation capacities are listed in Appendix A. Total load and generation capacity are therefore 23.8 GW and 27.4 GW, respectively.

2. Line and Transformer Capacities

As line and transformer capacities are not available in the IEEE data-set, estimates are obtained based on the following procedure:

1. Estimate line or transformer capacity from Table 1 based on bus voltages
2. Increase IEEE line or transformer IEEE power-flow solution values (provided by IEEE dataset with all components intact) by 20%
3. Assume the higher of (1) and (2) to be the rated capacity of the line or transformer.

Bus voltage (kV)	Estimated line capacity (MW)	Estimated transformer capacity (MW) [use high-voltage bus]
6.6	10	10
13.8	10	10
20.0	10	10
66.0	50	50
115.0	150	150
138.0	200	200
230.0	500	500
345.0	1,000	1,000

Table 1. Estimated transformer and line capacities based on bus voltages.

E. RECOP OUTPUT

To illustrate the typical output from RECOP, we assume a hypothetical scenario in which five buses and five lines of the 300-bus test network are damaged. Two of these damaged lines require a spare for repair, which we call "HVT." We suppose that no HVT is kept in inventory and that one unit can be delivered during the fifth time-period while a second unit can be delivered during the seventh. Each time-period lasts a day. Work-crews of types *r1, r2* and *r3* are available to carry out the repair work. These crews work in shifts and one team of *r1* and *r2* and two teams of *r3* are available at any time during a 24-hour time-period. Figures 1 and 2 display results.

<table>
<tr><th rowspan="2">Damaged component</th><th colspan="9">Time period (day)</th></tr>
<tr><th>1</th><th>2</th><th>3</th><th>4</th><th>5</th><th>6</th><th>7</th><th>8</th><th>9</th></tr>
<tr><td>Bus 37</td><td>r2</td><td>✓</td><td>✓</td><td>✓</td><td>✓</td><td>✓</td><td>✓</td><td>✓</td><td>✓</td></tr>
<tr><td>Bus 41</td><td></td><td>r2</td><td>✓</td><td>✓</td><td>✓</td><td>✓</td><td>✓</td><td>✓</td><td>✓</td></tr>
<tr><td>Bus 44</td><td></td><td></td><td>r2</td><td>✓</td><td>✓</td><td>✓</td><td>✓</td><td>✓</td><td>✓</td></tr>
<tr><td>Bus 51</td><td></td><td></td><td></td><td>r2</td><td>✓</td><td>✓</td><td>✓</td><td>✓</td><td>✓</td></tr>
<tr><td>Bus 92</td><td colspan="3">r3</td><td>✓</td><td>✓</td><td>✓</td><td>✓</td><td>✓</td><td>✓</td></tr>
<tr><td>Line (3, 4)</td><td></td><td></td><td></td><td></td><td></td><td></td><td colspan="2">r3 + HVT</td><td>✓</td></tr>
<tr><td>Line (41, 49)</td><td></td><td></td><td></td><td colspan="3">r1</td><td>✓</td><td>✓</td><td>✓</td></tr>
<tr><td>Line (42, 46)</td><td colspan="3">r1</td><td>✓</td><td>✓</td><td>✓</td><td>✓</td><td>✓</td><td>✓</td></tr>
<tr><td>Line (45, 46)</td><td></td><td></td><td></td><td></td><td colspan="2">r3 + HVT</td><td>✓</td><td>✓</td><td>✓</td></tr>
<tr><td>Line (61,62)</td><td></td><td></td><td></td><td></td><td>r2</td><td>✓</td><td>✓</td><td>✓</td><td>✓</td></tr>
</table>

Figure 1. Optimal repair schedule for hypothetical scenario. Cells with a diagonal line indicate components are awaiting repair. Labels *r1, r2 and r3* indicate the type of work-crew used to repair the damaged component. Label *HVT* shows the usage of consumable spare HVT. Cells with a check indicate that repairs have been completed on the component. For example, repair of damaged line (45, 46) starts during time-period five utilizing work-crew r3. It lasts for two time-periods and uses one unit of HVT.

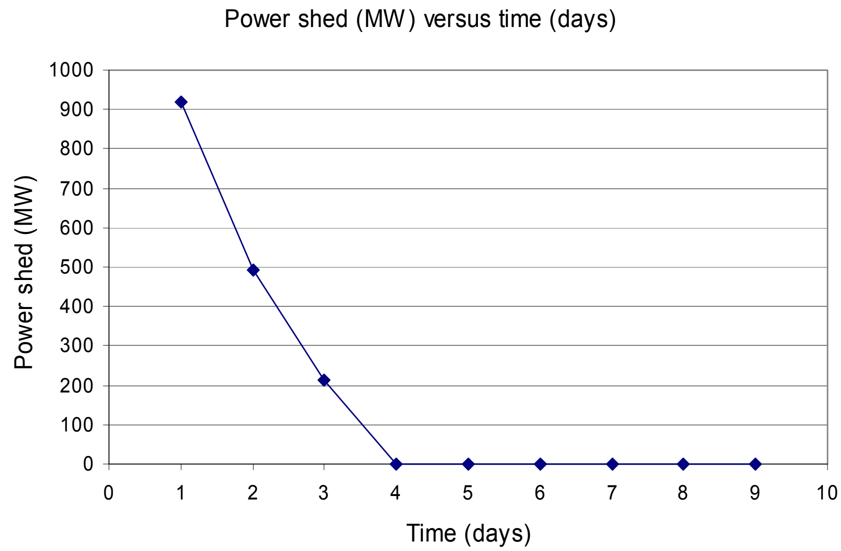


Figure 2. Chart showing the reduction in power-shedding as repair progresses with time. Note that the power shed drops to zero before repair completion due to excess capacity in the network.

III. RESULTS

A. OVERVIEW

We test RECOP on two scenarios that are both based on the IEEE 300-bus test network described previously.

The first scenario simulates relatively large-scale damage to the electrical grid caused by a storm or hurricane. Damage is assumed to be confined to exposed and vulnerable parts of the grid such as transmission towers and lines. We suppose that components and cables for the repair of damaged equipment are in adequate supply and do not place a constraint on the rate of the recovery process. Instead, scheduling of repair resources such as work-crew and repair machinery, while maintaining repair-precedence relationships, shall be the main focus.

The second scenario depicts a coordinated terrorist attack on the power grid. While the physical damage is likely to be less extensive than that caused by a storm, the impact can be larger as the terrorists can selectively destroy critical targets. These can include system components such as high-voltage transformers with long replacement lead-times. If the number of such destroyed components exceeds the number of available spares, a decision would have to be made regarding which parts of the grid are to be repaired first while waiting for the shortfall to be filled. In this scenario, we investigate the use of our model as a tool for repair-resource scheduling and repair prioritization when demand for replacement parts exceeds availability.

The objective of RECOP is to minimize the total cost of power shed over the recovery-time horizon, but determining the cost per kilowatt-hour of unserved demand for each of the loads on the grid is a complex matter: The cost varies by customer class type (commercial, industrial, residential etc), season, duration of outage and other factors. For the purpose of this thesis, we assume, for simplicity, an outage cost of $7,500/MWh for all loads. This outage cost is based on a report by ICF Consulting [2003] on the economic cost of the Northeastern blackout of August 2003.

The model is implemented using the General Algebraic Modeling System (GAMS) [2004] and solved with CPLEX 9.0 [2005.] Tests are carried out on a 2 GHz personal computer having 1 GB of RAM and running under the Microsoft Windows 2000 Professional operating system.

B. SCENARIO 1: HURRICANE DAMAGE

Utilities in hurricane-prone areas are usually well-equipped to deal with storm-induced damage, and recovery is often relatively swift. For example, Entergy Corporation [2005] was able to restore power to over three quarters of their customers affected by Hurricane Rita within two weeks. For our modest-sized test case, we suppose a recovery-time horizon of up to 15 days with each time period lasting one day.

To examine the impact of storms having varying severity, we randomly select 4% to 20% of the grid's buses and lines to be downed. Generators are assumed to be well-protected and cannot be damaged although they can be disconnected from the grid if their associated buses are damaged.

For network restoration, we suppose that the utility has a set of repair resources of types *r1*, *r2* and *r3*, each with different repair capabilities and availability that may change over time. We assume that each damaged component requires one to three units of any one type of resource for repair, and randomly assign a repair time of one to three days. Appendix B contains detailed lists of the damaged components, the resource availability over time, and also the time and the number of resource units required to repair each damaged component. Precedence relationships are also described.

All cases are configured in GAMS to solve until a relative optimality gap of 1% or less is achieved [GAMS/CPLEX 2003], or until the solver time reaches one hour, whichever occurs first. Table 2 presents results.

Percentage damaged (and number)	Optimality gap (%)	Maximum power shed (MW)	Energy shed (GWh)	Cost of unserved demand ($b)	Recovery duration (days)
4% (28)	1.0	2,713	174	1.3	8
8% (57)	1.0	4,912	418	3.1	9
12% (85)	1.8	6,998	670	5.0	9
16% (114)	3.7	9,788	999	7.5	11
20% (141)	7.0	11,937	1,376	10.3	12

Table 2. Impact of five different levels of damage on the grid due to a hurricane. "Percentage damaged" indicates the proportion of buses and lines in the grid that are damaged; the numbers in parentheses indicate the actual number of damaged components. Maximum power shed indicates load shedding immediately after the hurricane and assumes that the undamaged portion of the grid is stabilized and operational. Energy shed measures cumulative load lost until system repair is complete.

C. SCENARIO 2: TERRORIST ATTACK

We depict a coordinated terrorist attack on the power grid in this scenario. One of the easiest terrorist targets could be high-tension transmission towers and lines in remote areas. However, utilities regularly deal with damage to such infrastructure resulting from weather or malicious activities, and repairs can usually be accomplished quickly. We therefore suppose a smart enemy that will choose to target other, more critical components of the network in order to create more extensive disruptions. Some of these targets may include critical equipment with limited spares and long delivery lead-times. The Congressional Research Service [2004] wrote:

> Of the transmission system's physical infrastructure, high-voltage (HV) transformers are arguably the most critical component. Utilities rarely experience loss of an individual HV transformer, but recovery from such a loss takes months if no spare is available.

We therefore suppose a scenario in which the terrorists concentrate their attacks on high-voltage transformers and buses, and that the terrorists try to maximize the

disruption that they can cause by "shortlisting" as potential targets only those transformers with at least 400 MW of capacity, and buses with local generation capacity or loads. We assume that up to 20 randomly chosen buses or transformers from the terrorists' shortlist are damaged during a coordinated attack, and that each damaged bus or transformer requires one unit of replacement part, named *s1* and *s2*, for repair (or replacement), respectively. A limited number of *s1* and *s2* parts are kept as spares and are available for immediate use, but once these are consumed obtaining additional units incurs delivery or manufacturing lead time. The availability of these parts over time is summarized in Table 3. Appendix C contains detailed lists of the damaged components, the resource availability over time, and the time and resources required to repair each damaged component.

Component	Units in inventory	Units of components delivered at the start of each week									
		1	2	3	4	5	6	7	8	9	10
s1	2	0	0	2	2	4	0	0	0	0	0
s2	2	0	0	0	0	0	2	0	2	0	4

Table 3. Number of spare parts in inventory and the number of additional units that can be delivered over time.

We assume that repairs to a high-voltage transformer take four weeks once parts become available. This is loosely based on data provided by IEEE [1996], which states a repair time of 768 hours. For buses, we suppose that repairs will take two weeks. Finally, we assume that repair resources such as work-crew and machinery are available in abundance and do not place a constraint on the rate of recovery. This is reasonable as the number of damaged components is small and there is ample time to mobilize the necessary crews and machinery while awaiting arrival of the spares to commence repairs.

All test-cases are configured in GAMS to solve with a 1% relative optimality gap, or until the solver time reaches one hour, whichever occurs earlier. Table 4 presents the results.

Number of components damaged	Optimality gap (%)	Maximum power shed (MW)	Energy shed (GWh)	Cost of unserved demand ($b)	Recovery duration (weeks)
4	0.0	579	194	1.5	4
8	0.0	1,314	544	4.1	9
12	0.0	1,648	846	6.4	11
16	0.8	2,293	1,329	10.0	13
20	0.9	3,865	2,425	18.2	13

Table 4. Impact of five different levels of damage on the grid due to a terrorist attack. Maximum power shed indicates load shedding immediately after the terrorist attack and assumes that the undamaged portion of the grid is stabilized and operational. Energy shed measures cumulative load lost until system repair is complete.

The number of grid components that are damaged in the terrorist-attack scenario is a fraction of that assumed in our hurricane-damage scenario. Not surprisingly, the maximum power shed is also significantly lower at about 20% to 30% of the level due to hurricane damage. However, the impact of the terrorist attack in terms of energy shed (and therefore, economic cost) is much greater due to the prolonged recovery period. In these scenarios, the economic impact of a terrorist attack that damages just eight critical components exceeds that from a large storm that causes damage to eight percent of the grid. If the spares were delivered over a longer timeframe, measured in months rather than weeks, the economic consequences would be even larger. This highlights the potential impact of spares availability on grid recovery and underscores the need to keep a strategic inventory of critical components to facilitate quick restoration of the power grid. To this end, RECOP can be used as a tool to help evaluate the effectiveness of various mixes of inventories.

D. RECOP SOLUTION SPEED AND QUALITY

To obtain a good initial, upper bound to RECOP when solved directly with the MIP, we set its initial feasible integer solution using results from the heuristic solution procedure described in Section E. Tables 5 and 6 present the solution times and optimality gaps for out test cases.

Percentage of components damaged (and number)	Optimality gap (%)	Solver time (sec)
4% (28)	1.0	230
8% (57)	1.0	950
12% (85)	1.8	3,600
	1.5	7,200
16% (114)	3.7	3,600
	3.4	7,200
20% (141)	7.0	3,600
	6.1	7,200

Table 5. RECOP optimality gaps and solution times for the hurricane-damage scenarios.

Number of components damaged	Optimality gap (%)	Solver time (sec)
4	0.0	5
8	0.0	10
12	0.0	120
16	0.8	340
20	0.9	480

Table 6. RECOP optimality gaps and solution times for the terrorist-attack scenarios.

Note that for the terrorist-attack scenarios, which have relatively few binary decision variables, optimal or near-optimal solutions are obtained within 480 seconds. The NP-hard nature of RECOP becomes apparent in the hurricane-damage scenarios, where the optimality gap remains at 6.1% after two hours of solver time for the largest test-case with 20% of network components damaged.

In the following sections, we present two alternative methods to solving RECOP, at least approximately, and compare the solutions obtained using these methods to those obtained by solving the MIP directly.

E. HEURISTIC SOLUTION

After some manipulation, RECOP can be stated in standard form as:

$$
\begin{aligned}
\text{RECOP: } \min_{x,y} \quad & -c'x + cy \\
\text{s.t.} \quad & Ax \le r \\
& Bx + Cy = d \\
& x \text{ binary}, y \ge 0
\end{aligned}
$$

where y represents generation outputs, load-shedding, power flows and phase angles (original g, s^{-}, y and θ variables, respectively), and where x represents bus- and line-repair variables (original x^{n}, x, q^{n} and q variables, respectively.)

We can solve RECOP heuristically in a two-stage process. Firstly, we formulate and solve the following Heuristic sub-model-1 (H1-RECOP):

$$
\begin{aligned}
\text{H1-RECOP: } \max_{x} \quad & fx \\
\text{s.t.} \quad & Ax \le r \\
& x \text{ binary}
\end{aligned}
$$

where f represents heuristic weights for the relative importance of repairing each of the damaged buses and lines in the grid. For buses, we assign weights equal to the bus's local generation capacity plus the capacity of all its incident lines. Lines are assigned weights equal to their capacities. We then solve H1-RECOP to obtain $\hat{x}$.

Stage two of the heuristic simply solves:

$$
\begin{aligned}
\text{H2-RECOP: } \min_{y} \quad & cy \\
\text{s.t.} \quad & B\hat{x} + Cy = d \\
& y \ge 0
\end{aligned}
$$

H2-RECOP therefore obtains the minimum-cost, feasible power flow based on the recovery plan given by $\hat{x}$.

The heuristic solution procedure for RECOP is implemented in GAMS and solved using CPLEX. For all test cases, in both scenarios, feasible solutions are obtained within

10 seconds. We compare the quality of these solutions to those from the MIP formulation in Section G.

F. BENDERS DECOMPOSITION OF RECOP

RECOP has a bi-level structure that makes the model suitable for the application of Benders decomposition [Benders 1962]; this is a well-known technique for solving MIPs. The first level specifies the damage inflicted on the network, which is modeled through binary decision variables. The second level is a continuous network-flow model that determines the electrical power-flow through the damaged network. Since the second-level, continuous model is linear, it satisfies Benders' requirement for convexity in the subproblem.

Using the notation from the previous section, the subproblem (SP) for the Benders-decomposition implementation of RECOP may be written as:

$$\begin{aligned} \mathrm{SP}_k(\hat{x}_k):\ \min_{y}\ & cy && \text{[dual variables]} \\ \text{s.t. }\ & Cy = d - B\hat{x}_k && [\pi_{k+1}] \\ & y \geq 0 \end{aligned}$$

where π_{k+1} represents the dual variables of the corresponding constraints and k represents the iteration counter of the Benders Decomposition Algorithm (BDA), which is presented in Appendix D. We solve SP to obtain $\hat{\pi}_{k+1}$, the optimal values for π_{k+1}. The first iteration of BDA uses $\hat{x}$ obtained from H1-RECOP as input, in order to provide a good initial upper bound on the optimal objective value for the full problem.

The "master problem" (MP) for the Benders decomposition takes the form:

$$\begin{aligned} \mathrm{MP}_k:\ \min_{x,z}\ & z \\ \text{s.t. }\ & z \geq -c'x_i + \hat{\pi}_i(d - Bx)\ \ i = 1,2,\ldots,k \\ & Ax \leq r \\ & x \text{ binary} \\ & z \text{ unconstrained} \end{aligned}$$

where i indexes the constraints at a given iteration of BDA.

Figure 3 shows the sequence of lower and upper bounds (LB and UB, respectively) generated by BDA for the terrorist-attack scenario with eight components damaged. Convergence is rapid (21 iterations, 40 seconds), and the optimal recovery plan produced by BDA matches that produced by solving RECOP directly using the MIP. Figure 4 shows the bounds for the hurricane-damage scenario with 8% of components damaged. Solution time is one hour with 631 iterations. For our largest test case with 20% of components damaged, convergence is slow and a significant optimality gap of about 20% remains after one hour and 311 iterations; Figure 5 shows detailed results.

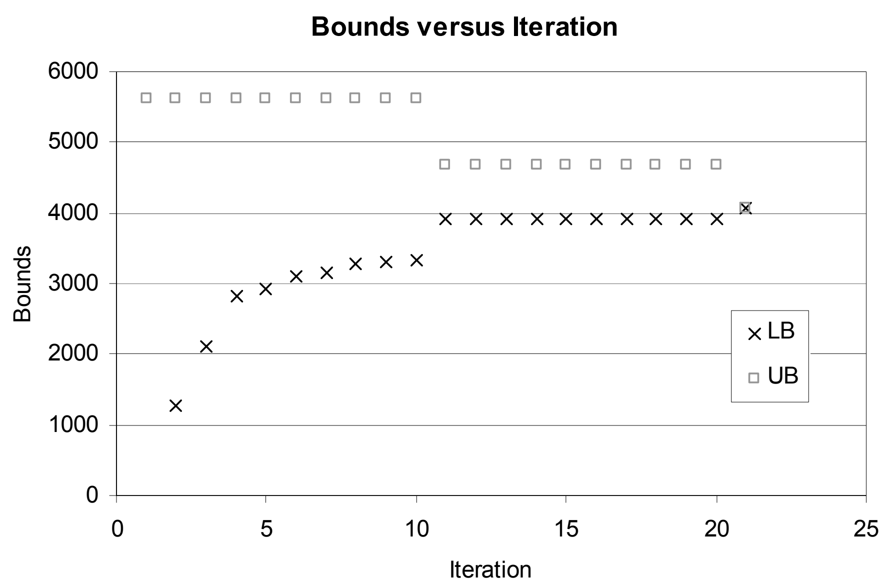


Figure 3. Convergence of BDA upper and lower bounds for terrorist-attack scenario with eight components damaged. Solution time is 40 seconds with 21 iterations. Initial lower-bounds are negative and have been truncated from the chart for clarity.

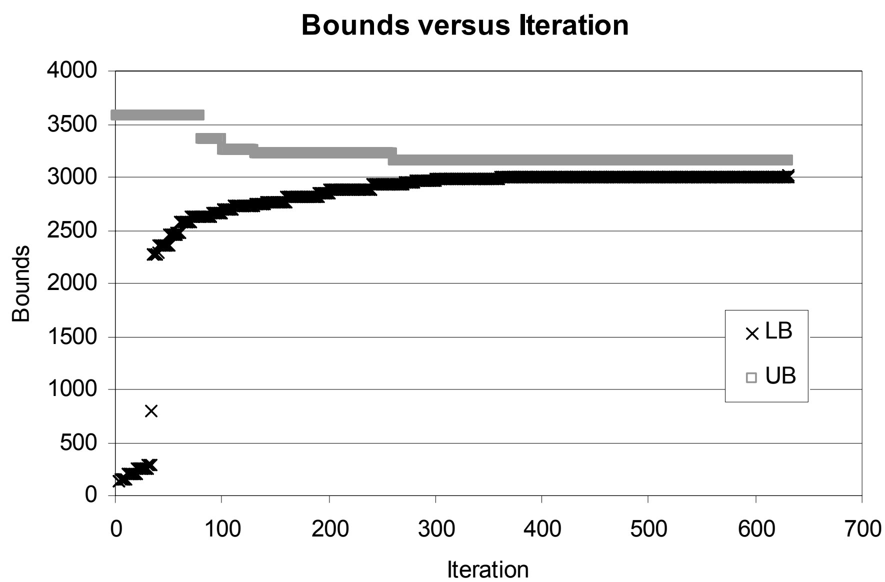


Figure 4. Convergence of BDA upper and lower bounds for hurricane-damage scenario with 8% of grid components damaged. Solution time is one hour with 631 iterations. Initial lower-bounds are negative and have been truncated from the chart for clarity.

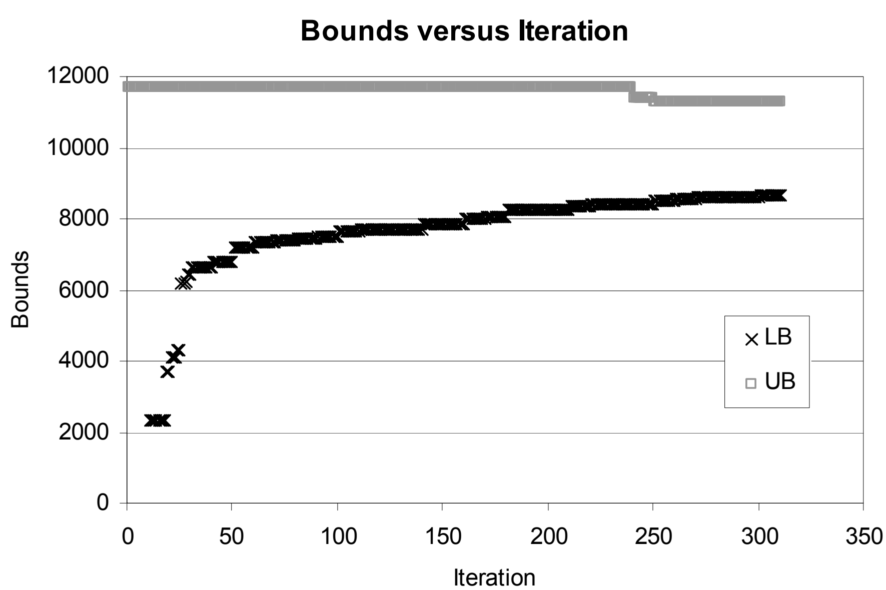
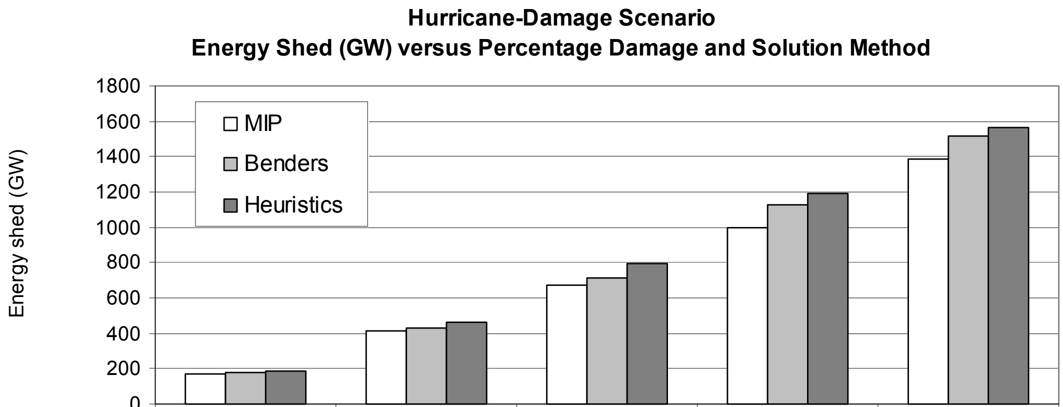


Figure 5. Convergence of BDA upper and lower bounds for hurricane-damage scenario with 20% of grid components damaged. Solution time is one hour with 311 iterations. Initial lower-bounds are negative and have been truncated from the chart for clarity.

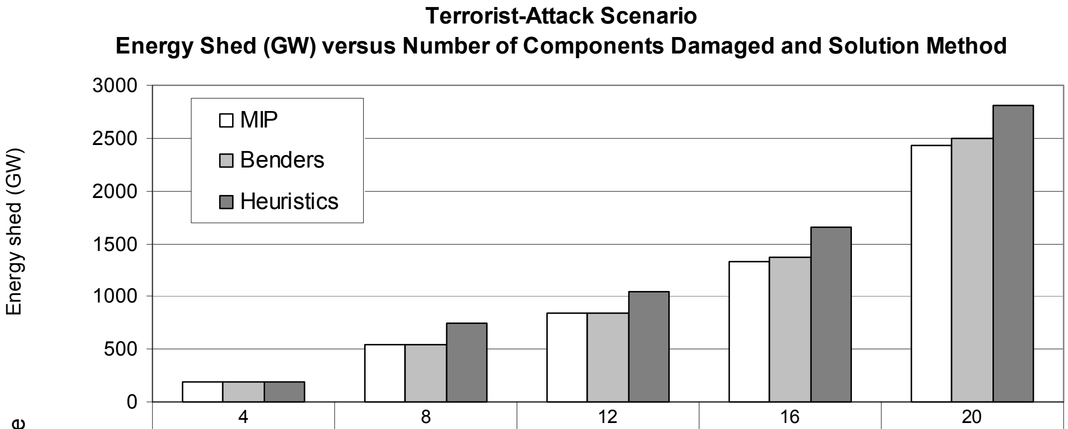
G. COMPARISON OF SOLUTION METHODS AND TIMES

This section compares the energy shed over the repair horizon, i.e., solution quality, and solution times, for solutions obtained for RECOP through the MIP, Benders decomposition and the heuristic. Figures 6 and 7 present the results for the hurricane-damage and terrorist-attack scenarios, respectively.



Solution time	4%	8%	12%	16%	20%
MIP	230 seconds	950 seconds	1 hour	1 hour	1 hour
Benders	1 hour	1 hour	1 hour	1 hour	1 hour
Heuristics	10 seconds	10 seconds	10 seconds	10 seconds	10 seconds

Figure 6. Energy shed and solution times for the hurricane-damage scenarios solved using MIP, Benders decomposition and a heuristic. Percentages along the horizontal axis indicate the proportion of grid components damaged.



Solution time	4	8	12	16	20
MIP	5 seconds	10 seconds	120 seconds	340 seconds	480 seconds
Benders	10 seconds	40 seconds	640 seconds	1 hour	1 hour
Heuristics	10 seconds	10 seconds	10 seconds	10 seconds	10 seconds

Figure 7. Energy shed and solution times for the terrorist-attack scenarios solved using MIP, Benders decomposition and a heuristic. Numbers along the horizontal axis indicate the number of grid components damaged.

It is apparent from Figures 6 and 7 that solving the MIP directly yields the best solution quality for both scenarios. Benders decomposition does not fare as well, taking at least as much time as the MIP to produce a solution of the same or inferior quality. Approximately solving RECOP by the heuristic fares worst in terms of solution quality. However, the heuristic achieves a good feasible solution quickly: In the hurricane-damage scenarios, heuristic solutions obtained in ten seconds are within 10% of that obtained by solving the MIP directly for an hour. The gap between the heuristic and MIP solution methods widens to a maximum of 40% in the terrorist-attack scenarios, where relatively few components are damaged. In these cases, the problem is small enough to be solved efficiently by the MIP. But, given an electrical grid that has thousands of buses, instead of the 300 in our test network, the heuristic solution procedure may be the most viable option in practice.

IV. CONCLUSIONS AND FUTURE WORK

This thesis has formulated and solved a mixed-integer RECovery OPtimization program (RECOP) to plan the recovery of an electrical power grid that has been damaged by a natural disaster or terrorist attack. The recovery plan takes into account the availability of repair resources and spares, and schedules the allocation of both in order to minimize the amount of unmet demand for energy over the planning horizon.

We implement RECOP in GAMS and use CPLEX as the solver. An electrical grid based on IEEE's 300-bus transmission network is used for testing. To simulate varying degrees of damage to the network, we choose up to 20% of the grid's buses, lines and transformers to be placed out of service. We demonstrate that RECOP can produce a recovery plan to minimize power-shedding costs that is within 7% of optimal within an hour. This result is obtained on a 2 GHz personal computer with 1 GB of RAM.

Additionally, we explore using a heuristic and Benders decomposition to solve RECOP, at least approximately. In all test cases, Benders decomposition takes at least as much time as the MIP requires to produce a solution of the same or inferior quality. For the terrorist-attack scenarios, where relatively few components are damaged, the heuristic solutions are up to 40% worse in terms of energy shed compared to the MIP solutions. The difference in performance between these two solution methods narrows in hurricane-damage scenarios, however, where more components are damaged. In these cases, each heuristic solution obtained in ten seconds is within 10% of the solution obtained by solving the MIP, given a one-hour time limit. This shows the potential of the heuristic solution method for larger-sized problems to obtain "reasonable" solutions within a short period of time. However, more computational trials using randomized sets of damaged buses and lines are required to understand the impact of input data on the variability of RECOP run times and the quality of heuristic solutions.

We demonstrate that, although the number of grid components that might be damaged in a coordinated terrorist attack may be relatively small compared to the number damaged by a hurricane, the impact in terms of energy shed and economic costs could be larger. This is true because terrorists can selectively target critical components that

cannot be replaced immediately due to delivery and manufacturing lead-times for replacement parts. This point highlights the potential impact of spares availability on grid recovery and underscores the need to keep a strategic inventory of critical components to facilitate quick restoration of the power grid. To this end, RECOP can be used as a tool to help evaluate the effectiveness of various mixes of inventories.

Areas for further research include:

1. Refining the Benders decomposition algorithm to improve its computational speed and quality.

2. Developing more advanced heuristics to improve solution quality. This may include determining better weights to prioritize the recovery of damaged components, and an iterative solution approach to repeatedly refine the recovery plan based on previous solutions.

3. Incorporating alternative objective functions in RECOP. Currently, the aim is to minimize the cost of energy shed over the recovery time-horizon. Other plausible objectives could be to minimize the recovery time or to ensure that, for example, 90% of affected customers have their power restored within a certain period of time. Additionally, if a base load of critical users exist, the model could be modified to plan the recovery process such that these users have their power restored first, before the scope of the recovery work is expanded to other affected parts of the grid.

4. Testing on real-world problems.

5. Extending the basic methodology to recovery-planning for other critical infrastructure systems (e.g., a road network following an earthquake.)

6. Extending the model to incorporate dependencies between infrastructures. For example, if both a road network and a power grid in an area are damaged, repairs to certain damaged components of the grid may not be able to commence until road access to these components is restored.

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APPENDIX A. IEEE 300-BUS NETWORK GENERATION CAPACITY

The table below lists the changes made to the generation capacities at some of the buses of the IEEE [1993] 300-bus network.

Bus	Original generation capacity (MW)	Revised generation capacity (MW)
37	0	500
41	0	300
51	5	150
108	117	150
126	0	500
128	0	800
130	0	1,000
143	696	1,000
3054	50	90

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APPENDIX B. DATA FOR HURRICANE-DAMAGE SCENARIOS

A. REPAIR-RESOURCE AVAILABILITY

Ten units each of *r1*, *r2* and *r3* are introduced into the pool of available repair-resources at the start of the first, fourth and eighth time-periods, respectively.

B. LIST OF DAMAGED BUSES AND LINES AND REPAIR PRECEDANCE RELATIONSHIPS

1. Scenario with 4% of Components Damaged

Damaged buses: 1, 2, 3, 4, 7, 17, 62, 69, 77, 79, 80 and 130.

Damaged lines (i, j): (1, 5), (2, 3), (2, 8), (4, 16), (5, 9), (7, 131), (8, 11), (8, 14), (9, 11), (14, 15), (15, 16), (15, 17), (16, 42), (89, 91), (90, 92) and (92, 103).

Pairs of damaged buses $[i, i']$ where bus i must be repaired before starting repair on bus i': [1, 2], [2, 3] and [3, 4].

Pairs of damaged lines $[(i, j), (i', j')]$ where line (i, j) must be repaired before starting repair on line (i', j'): [(1, 5), (2, 3)], [(2, 3), (2, 8)] and [(2, 8), (4, 16)].

2. Scenario with 8% of Components Damaged

Damaged buses: 1, 2, 3, 4, 7, 9, 17, 19, 20, 21, 22, 62, 69, 77, 79, 80, 125, 126, 130, 144, 151, 158, 167 and 170.

Damaged lines (i, j): (1, 5), (2, 3), (2, 8), (3, 19), (4, 16), (5, 9), (7, 131), (8, 11), (8, 14), (9, 11), (10, 11), (14, 15), (15, 16), (15, 17), (15, 37), (16, 42), (40, 48), (49, 51), (89, 91), (90, 92), (91, 94), (91, 97), (92, 103), (92, 105), (123, 124), (126, 157), (126, 158), (126, 169), (127, 168), (129, 130), (130, 132), (130, 168) and (158, 160).

Pairs of damaged buses $[i, i']$ where bus i must be repaired before starting repair on bus i': [1, 2], [2, 3], [3, 4], [4,7], [20, 21] and [22, 125].

Pairs of damaged lines $[(i, j), (i', j')]$ where line (i, j) must be repaired before starting repair on line (i', j'): [(1, 5), (2, 3)], [(2, 3), (2, 8)], [(2, 8), (4, 16)], [(2, 3), (3, 19)], [(3, 19), (4, 16)] and [(3, 19), (15, 37)].

3. Scenario with 12% of Components Damaged

Damaged buses: 1, 2, 3, 4, 5, 6, 7, 9, 13, 17, 19, 20, 21, 22, 34, 35, 62, 69, 77, 79, 80, 123, 124, 125, 126, 130, 134, 135, 144, 151, 158, 167, 170, 190, 193 and 197.

Damaged lines (i, j): (1, 5), (2, 3), (2, 8), (3, 19), (4, 16), (5, 9), (7, 131), (8, 11), (8, 14), (9, 11), (10, 11), (14, 15), (15, 16), (15, 17), (15, 37), (16, 42), (40, 48), (47, 113), (48, 107), (49, 51), (89, 91), (90, 92), (91, 94), (91, 97), (92, 103), (92, 105), (122, 125), (122, 157), (123, 124), (126, 127), (126, 157), (126, 158), (126, 169), (127, 134), (127, 168), (128, 133), (129, 130), (130, 132), (130, 168), (133, 137), (135, 136), (154, 156), (155, 161), (157, 159), (158, 160), (163, 164), (167, 169), (194, 219) and (195, 219).

Pairs of damaged buses $[i, i']$ where bus i must be repaired before starting repair on bus i': [1, 2], [2, 3], [3, 4], [4,7], [19, 20], [20, 21], [21, 22.], [125, 126] and [126, 130].

Pairs of damaged lines $[(i, j), (i', j')]$ where line (i, j) must be repaired before starting repair on line (i', j'): [(1, 5), (2, 3)], [(2, 3), (2, 8)], [(2, 8), (4, 16)], [(2, 3), (3, 19)], [(3, 19), (4, 16)], [(3, 19), (15, 37)], [(15, 37), (40, 48)], [(133, 137), (154, 156)] and [(133, 137), (155, 161)].

4. Scenario with 16% of Components Damaged

Damaged buses: 1, 2, 3, 4, 5, 6, 7, 9, 13, 17, 19, 20, 21, 22, 27, 34, 35, 62, 69, 77, 79, 80, 97, 123, 124, 125, 126, 130, 133, 134, 135, 136, 137, 144, 151, 158, 167, 170, 172, 173, 174, 181, 182, 186, 187, 190, 193 and 197.

Damaged lines (i, j): (1, 5), (2, 3), (2, 8), (3, 19), (4, 16), (5, 9), (7, 131), (8, 11), (8, 14), (9, 11), (10, 11), (14, 15), (15, 16), (15, 17), (15, 37), (16, 42), (40, 48), (45, 74), (47, 113), (48, 107), (49, 51), (89, 91), (90, 92), (91, 94), (91, 97), (92, 103), (92, 105), (122, 125), (122, 123), (122, 157), (123, 124), (126, 127), (126, 157), (126, 158), (126, 169), (127, 134), (127, 168), (128, 133), (129, 130), (130, 132), (130, 168), (133, 137), (134, 135), (135, 136), (136, 137), (136, 152), (145, 146), (145, 149), (152, 153), (154, 156), (154, 183), (155, 161), (157, 159), (158, 160), (162, 164), (162, 165), (163, 164), (165, 166), (167, 169), (184, 185), (190, 231), (194, 219), (195, 212), (195, 219), (197, 198) and (197, 211).

Pairs of damaged buses $[i, i']$ where bus i must be repaired before starting repair on bus i': [1, 2], [2, 3], [3, 4], [4,7], [19, 20], [20, 21], [125, 126], [126, 130], [134, 135], [135, 136], [136, 137] and [190, 193].

Pairs of damaged lines [(i, j), (i', j')] where line (i, j) must be repaired before starting repair on line (i', j'): [(1, 5), (2, 3)], [(2, 3), (2, 8)], [(2, 8), (4, 16)], [(2, 3), (3, 19)], [(3, 19), (4, 16)], [(3, 19), (15, 37)], [(15, 37), (40, 48)], [(133, 137), (154, 156)], [(133, 137), (155, 161)], [(145, 146), (154, 156)], [(154, 156), (157, 159)], [(154, 156), (165, 166)] and [(190, 231), (194, 219)].

5. Scenario with 20% of Components Damaged

Damaged buses: 1, 2, 3, 4, 5, 6, 7, 9, 13, 17, 19, 20, 21, 22, 27, 34, 35, 62, 69, 77, 79, 80, 97, 123, 124, 125, 126, 130, 133, 134, 135, 136, 137, 144, 151, 158, 167, 170, 172, 173, 174, 181, 182, 186, 187, 190, 193, 197, 198, 201, 202, 204, 206, 207, 209, 211, 212, 222, 236 and 241.

Damaged lines (i, j): (1, 5), (2, 3), (2, 8), (3, 19), (4, 16), (5, 9), (7, 131), (8, 11), (8, 14), (9, 11), (10, 11), (14, 15), (15, 16), (15, 17), (15, 37), (16, 42), (40, 48), (45, 74), (47, 113), (48, 107), (49, 51), (89, 91), (90, 92), (91, 94), (91, 97), (92, 103), (122, 123), (122, 125), (122, 157), (123, 124), (126, 127), (126, 157), (126, 158), (126, 169), (127, 134), (127, 168), (128, 133), (129, 130), (130, 132), (130, 168), (133, 137), (134, 135), (135, 136), (136, 137), (136, 152), (145, 146), (145, 149), (152, 153), (154, 156), (154, 183), (155, 161), (157, 159), (158, 160), (162, 164), (162, 165), (163, 164), (165, 166), (167, 169), (184, 185), (190, 231), (194, 219), (195, 212), (195, 219), (197, 198), (197, 211), (198, 203), (201, 204), (203, 211), (204, 205), (206, 207), (206, 208), (212, 215), (213, 214), (214, 215), (216, 217), (219, 237), (220, 221), (242, 245), (242, 247), (244, 246) and (245, 247).

Pairs of damaged buses [i, i'] where bus i must be repaired before starting repair on bus i': [1, 2], [2, 3], [3, 4], [4,7], [13, 17], [19, 20], [20, 21], [22, 125], [123, 124], [125, 126], [126, 130], [134, 135], [135, 136], [172, 173] and [172, 174].

Pairs of damaged lines [(i, j), (i', j')] where line (i, j) must be repaired before starting repair on line (i', j'): [(1, 5), (2, 3)], [(1, 5), (133, 137)], [(2, 3), (2, 8)], [(2, 8), (4, 16)], [(2, 3), (3, 19)], [(3, 19), (15, 37)], [(4, 16), (5, 9)], [(15, 37), (40, 48)], [(133, 137), (135, 136)], [(133, 137), (154, 156)], [(145, 146), (154, 156)], [(154, 156), (157, 159)], [(162, 164), (162, 165)], [(162, 164), (163, 164)] and [(190, 231), (194, 219)].

C. BUS REPAIR-RESOURCE REQUIREMENTS AND REPAIR TIMES

The table below shows the respective number of units of repair-resource *r1*, *r2* or *r3* required to repair each damaged bus, if that resource type is utilized for the repair, and the corresponding repair times.

Bus	Number of units of *r1*, *r2* and *r3* required for repair, if *r1*, *r2* or *r3* are selected, respectively			Repair time if using *r1*, *r2* or *r3* for repair (days)		
	r1	*r2*	*r3*	*r1*	*r2*	*r3*
1	1	2	3	3	2	1
2	1	2	3	3	2	1
3	1	2	3	3	2	1
4	1	2	3	3	2	1
5	1	2	3	3	2	1
6	1	2	3	3	2	1
7	1	2	3	3	2	1
9	1	2	3	3	2	1
13	1	1	1	1	1	1
17	1	1	1	1	1	1
19	1	1	1	1	1	1
20	1	1	1	1	1	1
21	3	1	2	2	1	3
22	3	1	2	2	1	3
27	3	1	2	2	1	3
34	3	1	2	2	1	3
35	3	1	2	2	1	3
62	1	2	3	3	2	1
69	1	2	3	3	2	1
77	1	1	1	1	1	1
79	1	1	1	1	1	1
80	1	1	1	1	1	1

Bus	Number of units of *r1*, *r2* and *r3* required for repair, if *r1*, *r2* or *r3* are selected, respectively			Repair time if using *r1*, *r2* or *r3* for repair (days)		
	r1	*r2*	*r3*	*r1*	*r2*	*r3*
97	3	1	2	2	1	3
123	1	2	3	3	2	1
124	1	2	3	3	2	1
125	1	2	3	3	2	1
126	1	2	3	3	2	1
130	1	1	1	1	1	1
133	1	1	1	1	1	1
134	1	1	1	1	1	1
135	1	1	1	1	1	1
136	1	1	1	1	1	1
137	3	1	2	2	1	3
144	3	1	2	2	1	3
151	3	1	2	2	1	3
158	3	1	2	2	1	3
167	1	2	3	3	2	1
170	1	2	3	3	2	1
172	1	2	3	3	2	1
173	1	2	3	3	2	1
174	1	2	3	3	2	1
181	1	1	1	1	1	1
182	1	1	1	1	1	1
186	3	1	2	2	1	3
187	3	1	2	2	1	3
190	3	1	2	2	1	3
193	3	1	2	2	1	3
197	3	1	2	2	1	3
198	3	1	2	2	1	3

Bus	Number of units of *r1*, *r2* and *r3* required for repair, if *r1*, *r2* or *r3* are selected, respectively			Repair time if using *r1*, *r2* or *r3* for repair (days)		
	r1	*r2*	*r3*	*r1*	*r2*	*r3*
201	3	1	2	2	1	3
202	3	1	2	2	1	3
204	3	1	2	2	1	3
206	3	1	2	2	1	3
207	3	1	2	2	1	3
209	3	1	2	2	1	3
211	3	1	2	2	1	3
212	3	1	2	2	1	3
222	1	2	3	3	2	1
236	3	1	2	2	1	3
241	3	1	2	2	1	3

D. LINE REPAIR-RESOURCE REQUIREMENTS AND REPAIR TIMES

The table below shows the respective number of units of repair-resource *r1*, *r2* or *r3* required to repair each damaged line, if that resource type is utilized for the repair, and the corresponding repair times.

Line	Number of units of *r1*, *r2* and *r3* required for repair, if *r1*, *r2* or *r3* are selected, respectively			Repair time if using *r1*, *r2* or *r3* for repair (days)		
	r1	*r2*	*r3*	*r1*	*r2*	*r3*
1, 5	3	2	1	1	2	3
2, 3	3	2	1	1	2	3
2, 8	3	2	1	1	2	3
3, 19	1	1	1	1	2	3
4, 16	1	1	1	1	1	1
5, 9	1	1	1	1	1	1

Line	Number of units of *r1*, *r2* and *r3* required for repair, if *r1*, *r2* or *r3* are selected, respectively			Repair time if using *r1*, *r2* or *r3* for repair (days)		
	r1	*r2*	*r3*	*r1*	*r2*	*r3*
7, 131	1	1	1	1	1	1
8, 11	1	1	1	1	1	1
8, 14	2	1	3	1	1	1
9, 11	2	1	3	3	1	2
10, 11	2	1	3	3	1	2
14, 15	2	1	3	3	1	2
15, 16	2	1	3	3	1	2
15, 17	2	1	3	3	1	2
15, 37	2	1	3	3	1	2
16, 42	2	1	3	3	1	2
40, 48	3	2	1	1	2	3
45, 74	1	1	1	1	1	1
47, 113	1	1	1	1	1	1
48, 107	2	1	3	1	1	1
49, 51	2	1	3	3	1	2
89, 91	3	2	1	1	2	3
90, 92	3	2	1	1	2	3
91, 94	3	2	1	1	2	3
91, 97	3	2	1	1	2	3
92, 103	3	2	1	1	2	3
122, 123	2	1	3	3	1	2
122, 125	2	1	3	3	1	2
122, 157	2	1	3	3	1	2
123, 124	2	1	3	3	1	2
126, 127	2	1	3	3	1	2
126, 157	2	1	3	3	1	2
126, 158	2	1	3	3	1	2

Line	Number of units of *r1*, *r2* and *r3* required for repair, if *r1*, *r2* or *r3* are selected, respectively			Repair time if using *r1*, *r2* or *r3* for repair (days)		
	r1	*r2*	*r3*	*r1*	*r2*	*r3*
126, 169	2	1	3	3	1	2
127, 134	2	1	3	3	1	2
127, 168	2	1	3	3	1	2
128, 133	2	1	3	3	1	2
129, 130	2	1	3	3	1	2
130, 132	2	1	3	3	1	2
130, 168	3	2	1	1	2	3
133, 137	3	2	1	1	2	3
134, 135	3	2	1	1	2	3
134, 184	1	1	1	1	2	3
135, 136	1	1	1	1	1	1
136, 137	1	1	1	1	1	1
136, 152	1	1	1	1	1	1
145, 146	2	1	3	3	1	2
145, 149	2	1	3	3	1	2
152, 153	2	1	3	3	1	2
154, 156	2	1	3	3	1	2
154, 183	2	1	3	3	1	2
155, 161	2	1	3	3	1	2
155, 164	2	1	3	3	1	2
157, 159	2	1	3	3	1	2
158, 159	2	1	3	3	1	2
158, 160	2	1	3	3	1	2
162, 164	2	1	3	3	1	2
162, 165	2	1	3	3	1	2
163, 164	2	1	3	3	1	2
165, 166	2	1	3	3	1	2

Line	Number of units of *r1*, *r2* and *r3* required for repair, if *r1*, *r2* or *r3* are selected, respectively			Repair time if using *r1*, *r2* or *r3* for repair (days)		
	r1	*r2*	*r3*	*r1*	*r2*	*r3*
167, 169	2	1	3	3	1	2
184, 185	3	2	1	1	2	3
190, 231	1	1	1	1	2	3
194, 219	1	1	1	1	1	1
195, 212	2	1	3	1	1	1
195, 219	2	1	3	3	1	2
197, 198	2	1	3	3	1	2
197, 211	2	1	3	3	1	2
198, 203	2	1	3	3	1	2
201, 204	2	1	3	3	1	2
203, 211	2	1	3	3	1	2
204, 205	2	1	3	3	1	2
206, 207	2	1	3	3	1	2
206, 208	2	1	3	3	1	2
212, 215	2	1	3	3	1	2
213, 214	2	1	3	3	1	2
214, 215	2	1	3	3	1	2
216, 217	2	1	3	3	1	2
219, 237	2	1	3	3	1	2
220, 221	2	1	3	3	1	2
242, 245	3	2	1	1	2	3
242, 247	1	1	1	1	2	3
244, 246	1	1	1	1	1	1
245, 247	1	1	1	1	1	1

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APPENDIX C. DATA FOR TERRORIST-ATTACK SCENARIOS

A. REPAIR-RESOURCE AVAILABILITY

Six units of *r1* and five units of *r2* are introduced into the pool of available repair-resources at the start of the first time-period, and six units of *r3* are introduced during the start of the second period.

B. LIST OF DAMAGED BUSES AND LINES

1. Scenario with 4 Components Damaged

Damaged buses: 5 and 11.

Damaged lines (i, j): (6, 7), and (10, 11).

2. Scenario with 8 Components Damaged

Damaged buses: 5, 11, 17 and 37.

Damaged lines (i, j): (3, 4), (6, 7), (10, 11) and (23, 24).

3. Scenario with 12 Components Damaged

Damaged buses: 5, 11, 17, 37, 84 and 92.

Damaged lines (i, j): (3, 4), (6, 7), (10, 11), (23, 24), (35, 36) and (45, 46).

4. Scenario with 16 Components Damaged

Damaged buses: 5, 11, 17, 37, 84, 92, 108 and 124.

Damaged lines (i, j): (3, 4), (6, 7), (10, 11), (23, 24), (35, 36), (45, 46), (81, 88) and (85, 99).

5. Scenario with 20 Components Damaged

Damaged buses: 5, 11, 17, 37, 84, 92, 108, 124, 143 and 152.

Damaged lines (i, j): (3, 4), (6, 7), (10, 11), (23, 24), (35, 36), (45, 46), (81, 88), (85, 99), (117, 159) and (133, 171).

C. BUS REPAIR-RESOURCE REQUIREMENTS AND REPAIR TIMES

The table below shows the respective number of units of repair-resource *r1*, *r2* or *r3* required to repair each damaged bus, if that resource type is utilized for the repair, and the corresponding repair times.

Bus	Number of units of *r1*, *r2* and *r3* required for repair, if *r1*, *r2* or *r3* are selected, respectively			Repair time if using *r1*, *r2* or *r3* for repair (days)		
	r1	*r2*	*r3*	*r1*	*r2*	*r3*
5	1	2	3	3	2	1
11	1	1	1	1	1	1
17	1	1	1	1	1	1
37	3	1	2	2	1	3
84	3	1	2	2	1	3
92	3	1	2	2	1	3
108	3	1	2	2	1	3
124	1	2	3	3	2	1
143	3	1	2	2	1	3
152	3	1	2	2	1	3

D. LINE REPAIR-RESOURCE REQUIREMENTS AND REPAIR TIMES

The table below shows the respective number of units of repair-resource *r1*, *r2* or *r3* required to repair each damaged line, if that resource type is utilized for the repair, and the corresponding repair times.

Line	Number of units of *r1*, *r2* and *r3* required for repair, if *r1*, *r2* or *r3* are selected, respectively			Repair time if using *r1*, *r2* or *r3* for repair (days)		
	r1	*r2*	*r3*	*r1*	*r2*	*r3*
3, 4	1	2	3	3	2	1
6, 7	1	1	1	1	1	1
10, 11	3	1	2	2	1	3
23, 24	3	1	2	2	1	3
35, 36	3	1	2	2	1	3
45, 46	1	1	1	1	1	1
81, 88	3	1	2	2	1	3
85, 99	3	1	2	2	1	3
117, 159	3	1	2	2	1	3
133, 171	1	2	3	3	2	1

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APPENDIX D. BENDERS DECOMPOSITION ALGORITHM

This section describes the Benders Decomposition Algorithm for solving RECOP. (The notation $\vartheta(\cdot)$ refers to the optimal objective-function value of the problem in the argument.) We use a modified form of Benders decomposition where we solve the relaxed master problem (RMP) in most iterations *k*, and solve the true master problem only once out of every ten iterations. As RMP solves much faster than MP, and the cuts derived from RMP are valid, this technique improves the convergence rate of the lower-bound. The algorithm follows.

Input: Initial solution $\hat{x}_0$ obtained from HS-RECOP, matrices *B* and *C*, vectors *c, c′* and *d*, and an optimality tolerance $\varepsilon > 0$.

Output: Recovery plan, x^* and associated power flows, generation levels, phase angles, and load shedding, jointly denoted as *y**.

1. Set the iteration counter *k* := 0, the lower bound (LB) := $-\infty$, and the upper bound (UB) := $+\infty$.

2. Solve $(\text{SP}_k(\hat{x}_k))$ for $\hat{y}_k$ and $\hat{\pi}_{k+1}$.

3. If *k* is a multiple of 10, let $\text{UB}_k := -c'\hat{x} + c\hat{y}_k$. If $\text{UB}_k \le \text{UB}$, then update the upper bound: UB := UB_k, and set $x^* := \hat{x}_k$.

4. If $\text{UB} - \text{LB} \le \varepsilon$, STOP; otherwise, continue to step 5.

5. Add the newly generated cut to (MP_{k+1}): $z \ge -c'x_{k+1} + \hat{\pi}_{k+1}(d - Bx_{k+1})$.

6. Set *k* := *k* + 1.

7. If *k* is a multiple of 10, solve (MP_k) for $\hat{x}_k$ and set $\hat{z}_k := \vartheta(\text{MP}_k)$; otherwise, solve the relaxed master problem (RMP_k) for $\hat{x}_k$ and set $\hat{z}_k := \vartheta(\text{RMP}_k)$.

8. If $\hat{z}_k \ge \text{LB}$, set $\text{LB} := \hat{z}_k$.

9. Return to step 2.

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